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INTERACTION OF PLANE NON-PARALLEL JETS

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The collision of near-wall jets on a smooth surface was examined by many authors [1, 2]. Within the framework of the theory of potential jet flows, the position of the interaction domain remains undetermined in this problem while the direction of the resultant jet is determined uniquely. Taking account of viscosity permits finding the position of the interaction domain [3]. It is important to note that integral conservation laws and the assumption that viscosity is not essential in the interaction domain are sufficient to establish the direction of the resultant jet. It is not necessary to know the pressure distribution on the surface here.

A feature of the problem of jet collision in the neighborhood of a corner is the fact that the integral conservation laws do not permit establishment of the direction of the resultant jet if the pressure distribution on the surface is not known in the interaction domain. The solution of this problem within the framework of the theory of potential jet flows is also not unique and does not permit the unique determination of the resultant jet direction.

A simple approximation solution of the problem of the collision of plane submerged incompressible near-wall jets in the neighborhood of a corner is represented in this paper within the framework of the infinitely thin jet approximation. Laminar and turbulent flow are examined in a quasilaminar approximation. It is noted that small changes in the interacting jet parameters can radically alter the resultant jet direction. The expediency is also shown of utilizing the infinitely thin jet approximation in other jet flow problems. A qualitative examination is performed of problems about jet impact in a corner and on the collision of several jets in space.

1. Let two plane near-wall submerged jets directed toward the line of plane intersection be propagated along two intersecting planes Ω_1 and Ω_2 (Fig. 1). The angle γ between the planes and the coordinates x_1 and x_2 are taken along the surfaces Ω_1 and Ω_2, respectively, along the normals to the line of intersection on which $x_1 = 0$, $x_2 = 0$. Let us assume that the domain of jet interaction lies near a point with coordinates $x_1 = 0$, $x_2 = 0$ and that the jet parameters at a certain distance from the corner are independent of the flow in the interaction domain. It is assumed that the jet sources are sufficiently remote from the interaction domain. We assume that the flow therein is stationary and has the following configuration: near the corner each of the jets is separated from the surface ($x_1{}^0$ and $x_2{}^0$ are the coordinates of the point of jet separation on the surfaces Ω_1 and Ω_2), the flow in front of the separation point is unperturbed, behind the separation point a domain is shaped with small changes in the pressure and low velocities which is considered stagnant, and one resultant jet is formed because of the collision. Assuming the jet parameters outside the interaction domain known for $x_1 > x_1{}^0$ and $x_2 > x_2{}^0$, we determine the resultant jet direction, the pressure in the stagnant zone, and its characteristic dimensions.

The question of the motion of the jet being separated must be examined to solve the formulated problem. Later the jet motion after separation is investigated in the infinitely thin jet (ITJ) approximation.

2. Let ξ be a vector line of the momentum field and outside this line the momentum equals zero. Let us write the momentum conservation law in the ξ, τ coordinate system

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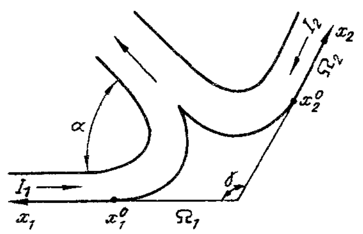

Fig. 1

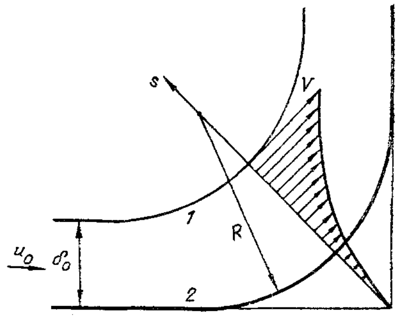

Fig. 2

associated with the line ξ (the coordinate ξ is measured along the line and τ, along the normal to it):

$$dI/d\xi = F_\xi,\ I/R = F_\tau, \tag{2.1}$$

where I is the momentum in the direction ξ; R is the radius of curvature of the line ξ; F_τ and F_ξ are the appropriate components of the external force vector acting per unit length.

If ξ separates two domains with different pressures and the pressure difference is of magnitude Δp, then

$$I/R = \Delta p. \tag{2.2}$$

A number of interesting properties results from the relationships (2.1) and (2.2): a) for $F_\xi = 0$ the quantity I = const along the line ξ; b) for $F_\tau = 0$ the line ξ is straight; c) for F_τ the line ξ is the arc of a circle (for instance, if ξ separates two domains with different pressures); d) the line ξ with momentum I can branch off along a tangent in lines with momenta I_1, I_2, ..., I_n, where $I_1 + I_2 + \ldots + I_n = I$. Let us note that the laws of ITJ motion follow from the momentum conservation law without involving the mass conservation law. The mass flow along the line ξ can change, which corresponds to the motion of a real jet.

Within the framework of the ITJ approximation the problem about the motion and interaction of submerged jets reduces to geometric problems. For example, during the collision of two jets on a plane with the momenta I_1 and I_2, each will stand off from the surface along the arc of a circle touching this surface. The resultant jet will emerge from the point of tangency of the circles to each other, directed along the tangent having the momentum $I_\Sigma = I_1 + I_2$. Since the difference between the pressure in the stagnant zone and the pressure in the remaining space is identical for each of the jets, then the radii R_1 and R_2 of the arcs of the circles are proportional to I_1 and I_2 of the respective jets. The momentum conservation law is satisfied automatically and the resultant jet direction is independent of the pressure in the stagnant zone.

The ITJ approximation is evidently applicable for small values of δ/R (δ is the characteristic transverse dimension of the jet). If $I = \delta U^2\rho$ (U is the characteristic velocity in the jet), then $\delta/R = \Delta p/\rho U^2$ from Eq. (2.2). Therefore, application of the approximation is related to the proposition that the pressure drop under whose action jet rotation occurs should be small as compared with the velocity head in the jet.

A comparison of the solution obtained by using this approximation with the exact solution of the problem of impact of a potential jet on an obstacle mounted at an angle $\gamma = \pi/2$ to the direction of unperturbed jet motion [1] is represented in Fig. 2 for a quantitative estimate of the suitability of the ITJ approximation. Here the pressure in the stagnant zone in the ITJ approximation equals the stagnation pressure. Then $\Delta p = \rho u_0{}^2/2$ and $R = 2\delta_0$ (u_0, δ_0 are the velocity and thickness of the unperturbed potential jet). The curve 1 is the outer boundary of the potential jet, 2 is the line of ITJ motion. The profile of the square of the absolute value of the velocity V in the potential jet is constructed along the angle bisector. It is seen that the pressure in the domain below the line 2 changes by approximately 10% from the velocity head of the unperturbed jet, while

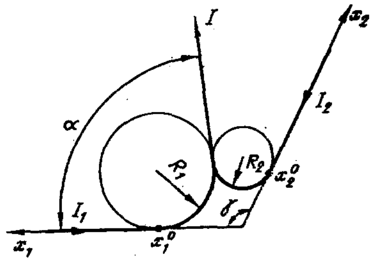

Fig. 3

it is considered constant in the ITJ approximation. The integral $I = \int_0^\infty \rho V^2 ds \simeq 0.77 I_0$. In the ITJ approximation this quantity is also considered constant and equal to I_0 (the coordinate s is measured along the angle bisector, $I_0 = \rho u_0^2 \delta_0$). Therefore, the ITJ approximation turns out to be completely applicable even when the pressure increase in the turning domain equals the velocity head of the unperturbed jet.

3. To solve the problem formulated in Sec. 1 about the collision of two near-wall jets in a corner in the ITJ approximation, it is necessary to construct two circles tangent to the sides of the corner γ and to each other. The radii of the circles are proportional to the momenta of the interacting jets (Fig. 3).

The condition for tangency of the circles to the sides of the corner and to each other can be written in the form

$$\begin{gathered} R_1(1+\cos\alpha) + R_2(\cos\gamma + \cos\alpha) = x_2^0 \sin\gamma, \\ R_1(1+\cos\alpha) + R_2[1+\cos(\gamma-\alpha)] = x_1^0 \sin\gamma + x_2^0 \sin(\gamma-\alpha), \end{gathered} \tag{3.1}$$

where R_1, R_2 are the radii of the circles proportional to the jet momenta; x_1^0, x_2^0 are coordinates of the points of jet separation or coordinates of the point of tangency of the circles and the sides of the corner, and α is the angle of the resultant jet direction (see Fig. 3).

It is seen from Eq. (3.1) that if R_1 and R_2 are given, then α is not determined single-valuedly since x_1^0 and x_2^0 are unknown. The boundary values are easily determined in the interval of possible α for given R_1 and R_2. Let $R_1 = R_2$, then for given γ the resultant jet direction is normal to the line x_2 and $\alpha = \gamma - \pi/2$ if the circle with radius R_1 is tangent to the lines x_1 and x_2 simultaneously. In the other extreme case $\alpha = \pi/2$. For example, for $\gamma = \pi/2$ and $I_1 = I_2$ the relationships (3.1) are satisfied for any α in the range $0 < \alpha < \pi/2$. Definite values of x_1^0 and x_2^0 correspond to each α. It is easy to note that in the general case $\alpha > \gamma$ or $\alpha < 0$ are possible. Then the resultant jet direction will intersect one of the coordinate lines and the flow configuration presumed cannot be realized.

Following the approach taken in the approximate theories of separation flows [4], let us take account of effects associated with viscosity for a single-valued choice of the solution satisfying Eq. (3.1). A local asymptotic theory of laminar near-wall jet separation as $\mathrm{Re} \to \infty$ is constructed in [2]. It is shown that a thin near-wall sublayer in which viscosity exists occurs in the neighborhood of the separation point in the jet. Under the action of the displacing effect of the viscous sublayer, the main part of the jet stands off from the surface and its motion is described by the Euler equations. Simple algebraic relationships in u_i, δ_i, ℓ, R, Δp are proposed in [3] for laminar or turbulent jets in a quasilaminar approximation, where u_i, δ_i, ℓ are the characteristic velocity, thickness, and longitudinal dimension of the viscous sublayer in the neighborhood of the separation point, R is the characteristic radius of jet rotation after separation, and Δp is the excess pressure in the zone behind the separation

$$\begin{gathered} u_i^2/2 = \Delta p,\ u_i/l = \nu/\delta_i^2,\ I/R = \Delta p,\ \delta_i/l^2 = 3/(4R), \\ u_i/u_m = \varphi(\delta_i/\delta_m); \end{gathered} \tag{3.2}$$

the function φ describes the velocity profile in the boundary layer in the unperturbed jet and is considered known, I is the unperturbed jet momentum, ν is the coefficient of kinematic turbulent velocity in the unperturbed jet at a distance $n = \delta_i$ from the wall, u_m is the maximum velocity in the jet, and δ_m is the distance from the wall at which the velocity takes on the value u_m.

The I, ν, u_m, δ_m in the relationships (3.2) are unperturbed jet parameters ahead of the separation point and depend on the coordinates of the separation point; consequently, by solving Eq. (3.2) for each of the interacting jets we obtain:

$$R_1 = R_1(x_1^0),\ R_2 = R_2(x_2^0),\ \Delta p_1(x_1^0) = \Delta p_2(x_2^0). \tag{3.3}$$

The relationships (3.3) together with Eq. (3.1) permit determination of x_1^0, x_2^0, R_1, R_2, $\Delta p_1 = \Delta p_2$ and the resultant jet direction (the angle α) if the dependences of the unperturbed jet parameters on the longitudinal coordinates x_1 and x_2 are known. The last relationship in Eq. (3.3) is the condition that the pressure in the closed zone bounded by the sides of the angle and by the jets being separated is constant.

As an illustration, let us examine the interaction of near-wall turbulent submerged jets by considering the jets self-similar ahead of the interaction domain. Then we write for the boundary layer in the near-wall jet [5]

$$u/u_m = (n/\delta_m)^{1/7} \tag{3.4}$$

(n is a coordinate measured from the surface across the jet). Therefore

$$\varphi(\delta_i/\delta_m) = (\delta_i/\delta_m)^{1/7}. \tag{3.5}$$

The quantity $I = \int_0^\infty u^2 dn$ ($\rho \equiv 1$) in Eq. (3.2) has the form

$$I = 4.76 u_m^2 \delta_m. \tag{3.6}$$

The turbulent friction in the boundary layer of the near-wall jet is $\tau = 0.01 u_m^2$ [5]. Then $\nu = \tau/(\partial u/\partial n)$ for $n = \delta_i$ and taking account of Eq. (3.4) we obtain

$$\nu = 0{,}07 \delta_i u_m^2/u_i. \tag{3.7}$$

Solving Eq. (3.2) with Eqs. (3.5)-(3.7) taken into account, we have

$$u_i = 0.81 u_m,\ \delta_i = 0{,}23\delta_m,\ \Delta p = 0.33 u_m^2,\ R = 14.4\delta_m,\ l = 2{,}1\delta_m.$$

Furthermore, using the relationship for u_m and δ_m [5] ($u_m = 3.5u_0/\sqrt{L/\delta_0}$, $\delta_m = 0.01L$ (u_0, δ_0 are the velocity and width in the initial jet section, and L is the distance from the initial jet section), we write Eq. (3.3) in the form

$$u_{01}^2/(L_0 - x_1^0) = u_{02}^2/(L_0 - x_2^0),\ R_i = 0.144\,(L_0 - x_i^0) \quad (i = 1, 2) \tag{3.8}$$

(L_0 is the distance between the initial jet section and the angle). It is considered that these distances, as well as the initial jet sections, are identical. The scale of variation of x_1^0 and x_2^0 are of the order of R. Let us introduce

$$\begin{gathered} \Delta = (u_{01} - u_{02})/2u^*,\ R_0 = 0.144 L_0, \\ \varepsilon = R_0/L_0,\ u^* = (u_{01} + u_{02})/2. \end{gathered} \tag{3.9}$$

Then taking Eq. (3.9) into account, we represent Eqs. (3.1) and (3.8) as

$$\begin{gathered} 1 + 2\cos\alpha + \cos\gamma = X_2^0 \sin\gamma, \\ 2 + \cos\alpha + \cos(\gamma - \alpha) = X_1^0 \sin\alpha + X_2^0 \sin(\gamma - \alpha), \\ \Delta = (\varepsilon/4)(X_2^0 - X_1^0)\ (X_i^0 = x_i^0/R_0,\ i = 1, 2) \end{gathered} \tag{3.10}$$

by considering $\varepsilon \ll 1$ and $\Delta \ll 1$ and keeping only the principal terms.

We obtain for the interaction of jets with nearby parameters ($\Delta \ll 1$) in an angle with $\gamma = \pi/2$ ($\varepsilon = 0.144$)

$$\sin(\pi/4 - \alpha) = 40\Delta. \tag{3.11}$$

It is seen from Eq. (3.11) that for $\Delta = 0;\ 0.07;\ -0.07$ $\alpha = \pi/4;\ 0;\ \pi/2$. Therefore, small changes in the initial velocity of each of the jets will result in a strong change in the resultant jet direction.

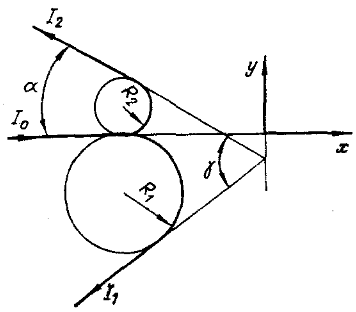

Fig. 4

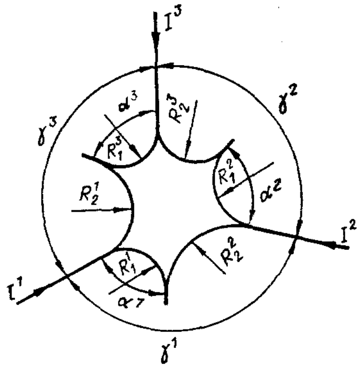

Fig. 5

A preliminary experimental investigation showed that for $\Delta = \pm 0.02$ the angle α took on the values 0 and $\pi/2$, respectively. However, for smaller values of Δ the resultant jet direction was unstable.

4. The ITJ approximations utilized in Sec. 3 can be useful for a qualitative analysis of certain other jet flows; for instance, in the problem of jet impact in a corner. Let a plane submerged jet be directed along the x axis parallel to the angle bisector and make impact on the intersecting surfaces forming the angle γ. The apex of the angle is at a point with coordinate y (Fig. 4).

The jet momentum equals I_0. Because of the collision the jet is separated into two that rotate along the arcs of circles with radii R_1 and R_2 and the momenta of these jets are I_1 and I_2, where $I_1 + I_2 = I_0$. A zone in which we consider the pressure constant and equal to $p_0 = p + \Delta p$ occurs in the interaction domain (p is the pressure outside the interaction domain). Then $R_i = I_i/\Delta p$ $(i = 1, 2)$. The force acting on the angle has the components $F_x = I_0(1 + \cos(\gamma/2))$, $F_y = \Delta p(R_1 - R_2)\sin(\gamma/2)$. Taking into account that $\alpha = \gamma/2$, the first equation in Eq. (3.1) can be written as

$$R_1 - R_2 = -y\cos(\gamma/2)/\cos^2(\gamma/4).$$

Then $F_y = \Delta p y \sin\gamma/[2\cos^2(\gamma/4)]$. It is seen that the force component F_y depends on the angle γ and on the coordinate y while F_x depends only on γ. If only a force caused by the jet impact acts on the angle in the direction y and the velocity of angle motion is small as compared with the fluid velocity in the jet, then the equation of angle motion in the direction y has the form

$$\ddot{y} \sim -y\sin\gamma/[2\cos^2(\gamma/4)].$$

Therefore, for $\gamma < \pi$ the angle motion will be oscillatory, where the frequency f of the oscillations depends on γ: $f \sim \sqrt{2\sin\gamma}/\cos(\gamma/4)$. The maximal frequency corresponds to $\gamma = 103.6°$. For $\gamma = \pi$ there will be no oscillations.

5. Another example of a problem in which the ITJ approximation can be useful is the problem of the collision of submerged jets in space. In the ITJ approximation a closed domain of elevated pressure should occur during jet interaction, whose boundaries are formed by arcs of circles tangent to each other that are convex toward the elevated pressure domain. Let N jets collide in a certain domain of space. By virtue of the mentioned geometric features of the elevated pressure domain boundary between the two adjacent jets, a jet should exist that is directed from the interaction zone, as is displayed in Fig. 5 in the example of the collision of three jets.

This means that within the framework of the approximation taken, the following assertion can be formulated: during interaction of plane jets in a space just as many jets will flow out of the interaction domain as flow into the interaction domain.

Let us number the jets flowing into the interaction domain from 1 to N. Let us denote the jet momenta by I^k $(1 \le k \le N)$, the angle between the k-th and (k + 1)-th jets by γ^k for $k \le N - 1$ and the angle between the N-th and first jet by γ^N as is shown in Fig. 5. In each angle γ^k $(1 \le k \le N)$ two relationships (3.1) can be written for $R_1{}^k$, $R_2{}^{k+1}$, x^{0k}, x^{0k+1}, α^k (see Fig. 5) for $k \le N - 1$ and for γ^N relative to $R_1{}^N$, $R_2{}^1$, $x_i{}^{0N}$, x^{01}, α^N. Moreover, $R_1^k + R_2^k = I^k/\Delta p$ (Δp is the excess pressure in the interaction zone). Therefore, 3N relationships in the 4N + 1 unknowns R_1^k do not permit a single-valued description of the interaction pattern, in particular, finding the directions of the jets flowing out of the interaction zone. An analogous result is obtained in the problem of the collision of two potential jets in a classical formula within the framework of TFKP [1, 6]. A sufficient number of papers (for example, [1, 7], etc.) are devoted to seeking the conditions governing the unique solution of this problem.

If the colliding jets are not potential, then it is natural to assume that the stagnation pressures of the current jets according to which separation of each of the interacting jets occurs are mutually equal and equal to the maximal pressure in the interaction zone (this condition is automatically satisfied for potential jets since the stagnation pressure of all the current jets is identical). If the stagnation pressure profiles in each of the interacting jets are known, then the stagnation pressure of the dividing jets of current p_0^k is a known function of R_1^k/R_2^k. Then N relationships

$$\Delta p = f_k(R_1^k/R_2^k),$$

can still be written, which will yield 4N relationships in 4N + 1 unknowns.

Therefore, it is sufficient to give the excess pressure, for example, in the interaction zone or the position of the dividing current jets in one of the interacting jets to select the unique solution in the ITJ approximation. Depending on the specific form of the velocity profiles of the interacting jets here and the angles γ^k, no current jet can be dividing, i.e., the system of equations has no solution for any value of Δp. In the author's opinion, the following is the most natural condition for the selection of the value of Δp: Δp should be the least of those possible.

It is seen from Fig. 5 that a situation is possible when the direction of one of the jets flowing out of the interaction domain will intersect one of the adjacent inflowing jets. Then, as for the collision of jets in an angle (Sec. 3), the flow configuration presumed cannot be realized.

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